



On the Optimality of the Competitive Process: Kimura's Theorem and Market Dynamics

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Synopsis: This paper considers the optimality properties of a market economy in terms of three propositions that evaluate the outcomes of and the process of competition between a population of firms working within a given economic environment. We show that when firms differ in more than one competitive characteristic then competition does not select in general the most efficient firm nor does it always result in increases in the average efficiency with which resources are utilized. Drawing upon a theorem of Kimura, however, we show that competition has the property of maximizing the rate of change of the average selective characteristics in the population. We conclude that a more nuanced appraisal of the institutions of the competitive process is surely necessary. From an evolutionary standpoint, the outcomes of competition are always contingent on the nature of the selection environment and the characteristics of the whole population of firms that are being selected.

Key words: competition, evolution, efficiency, optimality, growth

JEL classification: O33, L1

Introduction

The question of the optimality properties of an evolutionary process, if any exist, has long been a matter of keen interest to evolutionary biologists. Similarly, the question of the possible optimality properties of the market process has occupied the thinking of economists at least since the time of Adam Smith. Indeed, the connection he made in the *Wealth of Nations* between self-interest and benevolent market outcomes has been a central theme in economics ever since. The purpose of this paper is to bring together these two strands of thought by asking the question: 'What does economic competition optimize?'. This is not a trivial issue. Any answer informs the appraisal of the institutions of capitalism and has great significance for the conduct of public policy in regard of such matters as the efficacy of competition, the benefits from free trade, and wider processes of growth and development.

However, modern economics has generally approached this question from a decidedly non-evolutionary direction, that of the theory of market co-ordination in which the allocation of resources in equilibrium is said to be Pareto efficient. This body of analysis rests on two general propositions: that all agents are rational, and that all agents produce, exchange and consume within a framework of perfectly competitive markets in which no agent has any influence on the prices that co-ordinate plans to buy

and sell. In such states, no agent can be better off without some other agent being worse off. What is maximized, in a Pareto optimum, is the efficiency of the use of given resources to satisfy given needs at a point in time and over time.

In the evolutionary economic literature there is little to match this elegant body of analysis and for two reasons. Firstly, the rule-based agents generally assumed by evolutionary economists are no longer rational maximizers and this postulate, and its implication for behavior at the margin of choice, is crucial to the welfare propositions of orthodox theory. By abandoning the optimality of individual decisions, the foundation of Pareto welfare theory is eliminated at a stroke, a point made with great force by Lipsey & Lancaster (1956). They demonstrated, that if the Pareto conditions are absent in one market, then a 'second best' optimum is only attainable, in general, by departing from the first-order Pareto conditions in all other markets. Moreover, the magnitude and direction of these secondary departures cannot be described in terms of simple and sufficient *a priori* conditions to re-establish an optimum (Lipsey & Lancaster 1956, Ng 1979). Consequently, it has long been understood that the static competitive principles of the Pareto argument could only be applied with difficulty to an innovation based and restless capitalism (Stiglitz 1997). Indivisibilities in the generation of knowledge for innovation, the uncertainties that innovation entails, and difficulties in relation to property rights in new knowledge, create exactly the 'market failures' upon which the second best theorem thrives. Secondly, evolutionary economics replaces the postulate of a state of perfectly competition equilibrium with the study of a competitive process between rival and different firms and it is not proper to equate the description of a state with the analysis of a process (Witt 1999). More deeply than this, there is an important lacuna in the evolutionary argument, the lack of alternative principles for a normative assessment of the messy, imaginative and dynamic economics of creative destruction. To the extent that the evolutionary argument provides for a greater understanding of the restless nature of market capitalism in which innovation is the driving force for change, it is unfortunate that so little is said 'in judgement of progress' and of the institutional context, that generates economic change (Metcalfe 2001). The degree to which the competitive process leads in the direction of economic efficiency is a question of wide importance, yet one that remains to be answered satisfactorily (Hodgson 1993, Khalil 2001). By focusing on competition as a process that resolves differences between rival firms into patterns of changing resource utilization, an evolutionary approach can provide new insights into the working of the market mechanism and a new basis for the appraisal of economic institutions. I hasten to add that the general issue raised in this paper is not new. I only claim to bring it within a unified evolutionary framework.

In his famous defence of the profit maximization postulate in economics, Friedman (1953) argued that the selective forces associated with a *competitive process*, automatically lead to the survival of only the most efficient firms in a market and these will be the profit maximizing firms. Only profit maximizing behavior, therefore, need be taken into account when formulating if/then statements in economics, for all other models of behavior are transients in the market place. As several scholars have pointed out, this argument will not do. The process of competitive selection and its outcomes must be demonstrated not asserted (Koopmans 1957). More fundamentally, as Winter (1963)

conclusively argued, non-maximizing firms will outperform maximizing firms if they have better technologies or organizational structures. Competition selects for actions not decision processes. With the benefit of hindsight it appears that profit maximization is the wrong issue to focus the argument around; what matters for competition are the differences in behavior across firms, for then, even if they are maximal the different calculations are local not global. Friedman's conclusion only applies in a world of global, Olympian optimization in which the behavior of all firms is reduced to dull uniformity. This is not a world in which to make sense of innovation, nor the differences in knowledge and behavior on which it is premised.

The purpose of this paper is to contribute to this debate by evaluating three separate propositions about the efficiency of a market based evolutionary process. The first proposition is the evolutionary analogue to the static optimality claims of Pareto theory, to the effect that competition discovers those methods that ensure that goods and services are produced in the long-run with the most efficient techniques available. The second is a widely held belief about the efficacy of the competitive process and its scope for improving efficiency in resource utilization over time. The third is quite different in character, and we relate it to a theorem in the evolutionary literature that I call Kimura's theorem (Kimura 1958, Crow & Kimura 1970) a theorem that appears to be unknown in the evolutionary economics literature.

- Proposition A: competition, in the long run selects (discovers) the most efficient producer from among a population of rival producers of a homogenous commodity, with the corollary that all the less efficient producers are driven from the market;
- Proposition B: competition increases the average efficiency with which resources are utilized over time in an industry/market, with the corollary that the more efficient firms always grow more rapidly than do their less efficient rivals.

Propositions A and B are related but they are not identical, we will see that A can be false while B can be true. Conversely, if A is true, B must be correct at least for some stages in the evolution of an industry. However, the issue raised in relation to B, is whether it is true for all stages of that evolution. To untangle these two propositions is part of our task.

While the first two propositions will be familiar to any economist, the third is unlikely to be so since it is of a quite different kind. In 1958, the biologist Motoo Kimura published a highly original argument on the optimality of selection processes in a model of evolution.¹ In this paper, he demonstrated that in a population of entities varying with respect to their growth rates, the measure of fitness, the rate of change of the average growth rate of the corresponding population is maximized if and only if the dynamic rule that changes the relative importance of the different entities is a particular type of replicator process. Although Kimura applied his argument to growth rates, it is plausible to expect it to apply to the attributes of the population that determine those growth rates, and this is the problem addressed by proposition C.

- Proposition C: a competitive process generates the maximal possible rate of change in the average efficiency with which resources are utilized in an industry/market.

In applying Kimura's theorem to the competitive process, we uncover a dynamic question of great importance, 'Does market capitalism maximize the rate of change in the efficiency with which resources are used in the economy?' Notice that this proposition is quite independent of the previous two, in particular it is claimed to apply irrespective of whether proposition B is true.

These are the three propositions that motivate the following discussion. Taking all three together, they provide a powerful set of ideas to judge the working of the competitive market process in capitalist economies. In part two, we develop an evolutionary framework to evaluate the three propositions and, in part three, we carry out that evaluation.

A stylized model of competitive dynamics

For reasons that soon become clear, we will investigate a model of competitive selection in which firms differ in more than one selective characteristic. This model generates a set of if/then statements to link the growth rates of firms to their selective characteristics. Its purpose is to untangle a number of factors at play in a stylised market process with strong evolutionary credentials. We make a sharp distinction between the given and different properties of firms in a number of dimensions and the selective properties of markets, in particular, the product market and the capital market. This framework allows us to develop some taxonomy of the possible states a firm may occupy in the selection process. In so doing, we draw attention to the importance of the correlation between the selective characteristics of the population of firms for the rate and optimality of economic change. Readers familiar with evolutionary theory will notice immediately that it constitutes only 'half an argument' since nothing is said about the generation of the differences in the characteristics across the firms, nor how these differences might evolve under competitive pressure (Nelson & Winter 1982, Dosi 2000, Metcalfe 1998, 2001, Cohen & Malerba 2001). Nonetheless, half the argument is sufficient to make clear the fundamental issues.

The following account is only one of many equally valid ways of expressing the core features of a model of evolutionary competition. Those core features are defined in terms of the distribution of differences across firms in their competitive (selective) characteristics, and a market environment that evaluates those characteristics and connects the resultant differences in profitability to the differential growth of the firms (Nelson & Winter 1982, Silverberg & Verspagen 1998, Dosi 2000, Andersen 1994, Metcalfe 1998, Mazzucato 2000). Whatever the particular instantiation of the core theory the general outcome is the same, evolutionary competition.

However, when firms differ only with respect to the single selective characteristic, productive efficiency, the evaluation of our three propositions is unlikely to be compelling; indeed, it will verge on the trivial. Firms typically differ in multiple dimensions and the veracity of the three propositions has not been established for this general case. Moreover, the answer surely depends on the characteristics of the market environment in relation to its selective properties. As in evolutionary arguments in general, the

dynamics of change are driven both by the variety in the units of selection and by the properties of the selection environment. The model of competition is as follows.

A population of firms produce the same product, which is sold into a market growing at the constant, exogenous, positive compound rate, g_M . We recognize, of course, that the market growth rate typically declines over the life of an industry and that it can be expected to turn negative at some stage. Industries, like their constituent firms, do not live forever (Ormerod et al. 2001). Taking the growth rate as a positive constant is an expositional device, and, absent non-constant returns to scale in the cost functions for each firm, it is innocuous enough. Notice though, that while the argument generalizes to cases of zero and negative market growth, the analysis requires a number of modifications in these cases. Selection in declining markets is not the same as selection in expanding markets, expansion and contraction are not mirror images of one another.

Each firm is characterized by a bundle of decision routines and two sets of routines are highlighted in this account. The first set constitutes the routines in relation to the technology and organization of the firm, the rules that generate its efficiency level, and the unit cost level, h_i . Unit cost is the first selective characteristic, and each firm is assumed to have a different, given value for its unit costs. The fact that we do not discuss technical progress, or any other reasons for costs to vary over time, is again a convenient simplification not an issue of principle concerning ‘the second half of the argument’. The second bundle of routines relates to the dynamic investment behavior of the firm. Each firm has a rule to expand its productive capacity according to its current profitability and its given capital:output ratio. Let g_i be the rate of growth of capacity, and p_i the price set by the firm, then the investment rule is written as

$$\begin{aligned} g_i &= -\phi_i + \mu[p_i - h_i]; & (p_i - h_i > \phi_i/\mu) \\ &= 0 & (\textit{otherwise}) \end{aligned} \tag{1}$$

The ratio ϕ_i/μ measures the minimum unit profit margin for the firm, the cut off value below which it will not invest. The coefficient ϕ_i is a measure of the propensity of the firm to invest. Given profitability, a higher absolute value of ϕ_i indicates a lower propensity to invest in the sense that a higher profit margin is required to support a given growth rate. The coefficient μ is taken to be the same for all firms and it measures the sensitivity of the growth rate to the unit profit margin. We assume that all profits are distributed to shareholders and that a capital market funds all investment, and so we may interpret μ as a measure of the willingness of the capital market to fund the investment plans of any given firm. Thus (1) brings together the idiosyncratic differences across firms in their investment behavior and profitability, and the non-discriminating role of the capital market in relation to the supply of finance. It is apparent that (1) could be given several alternative interpretations but the one chosen will suffice for present purposes. If $\mu = 0$, the capital market is ‘closed’ for this population of firms, no funds are available for investment and the firms cannot grow their capacity whatever their profitability. If $\mu = \infty$, funds are available in unlimited

quantities, and the growth rate of a firm is not financially constrained. In this case, each firm would be obliged to set a price exactly equal to its unit costs. The following account focuses on the general case, $0 < \mu < \infty$. Notice that the rate of growth of capacity must be distinguished from the rate of growth of demand and output, although the pricing behaviors of the firms will tie the three growth rates together as we make clear below.

The population of firms is embedded in two other markets, an ‘inputs’ market, and a product market. The ‘inputs’ market supplies labor, materials and other means of production in unlimited quantities, at given prices. Consequently, inter firm differences in unit costs exactly reflect differences in their efficiency, as determined by their technological and organizational rules. The product market plays a more active role in the argument.

In the product market demand shifts between firms according to the prices they set. If consumers mix at random then the rate of growth of demand for the individual firm, g'_i , will depend on how the price it sets compares with the average of prices for all the firms, \bar{p} .

$$g'_i = g_M + \delta[\bar{p} - p_i] \quad (2)$$

The average price, \bar{p} requires careful definition, for the moment we simply note that it is defined over all the active firms in the market (Phelps & Winter 1970, Metcalfe 1998).

The product market selection coefficient, δ , plays an important role in the following, it is our measure of the information disseminating efficiency of the market institutions. If $\delta = \infty$, all firms must set the same price and we have a perfectly efficient market in which information about prices is diffused to all participants. If $\delta = 0$, each firm is an isolated monopolist and each may set a price independently from the other firms. In this case, we can say that each firm has its own particular market in which the consumers know only its price. In the following, we will be focused on the general case with $0 < \delta < \infty$, that is to say, a product market that is to a degree efficient but imperfectly so.

To complete the framework we need to attend to two further matters. The first is pricing behavior. Each firm sets its own price and it does so to co-ordinate the rate of growth of its own particular market with the rate of growth of its capacity to supply that market at fully capacity utilization. Consequently, demand growth, output growth and capacity growth are the same. We call these prices ‘normal’ prices. This device has the effect of suppressing important questions about price expectations and the consequences of capacity, demand and output following divergent paths. It is a powerful simplification as the reader will appreciate but to consider these deep matters would take us too far a field from the limited objectives we have set. It is clear, however, that the analysis of competition in non-normal conditions can result in the same kinds of ‘inefficient’ outcomes as are discussed below (Currie & Metcalfe 2001). The normal price a firm sets will depend on the nature of the markets and the prices set by rivals, as reflected in \bar{p} .² Of course, if the firm is to produce at all, we must have $p_i \geq h_i$, which is our viability condition.

Partitioning the Population

Based on these assumptions, we can always partition a given set of firms into a number of mutually exclusive economic categories, and the criteria on which this partitioning is based are indicated in Figure 1. As we shall show, the way in which the industry evolves depends on this partitioning, and how it varies over time. At the top level is the viability test, a firm is viable only if it can charge a price that can at least cover its unit costs of production ($p_i \geq h_i$), otherwise we consider that it is closed down ($p_i < h_i$). Those firms that are viable can be further partitioned into three groups according to a dynamic test, namely, the rate and manner in which they achieve growth of capacity and output. The important distinction here is between dynamic, stationary and marginal firms. Dynamic firms are firms that grow as the result of the interaction between (1) and (2). They set prices, which equate the rate of growth of capacity with the rate of growth of their particular market. By contrast, stationary firms are necessarily profitable but they have ceased to invest in capacity because their unit profit margin has reached the required cut-off level. Such a firm has a zero growth rate of output and, we assume, adjusts its price to maintain full utilization of its *given* capacity: that is to say, it sets a normal price which yields zero growth in demand, and this price is governed by (2) alone. Finally, we have the firms on the static viability margin, those stationary firms whose price has been driven into equality with their unit costs, who from that point

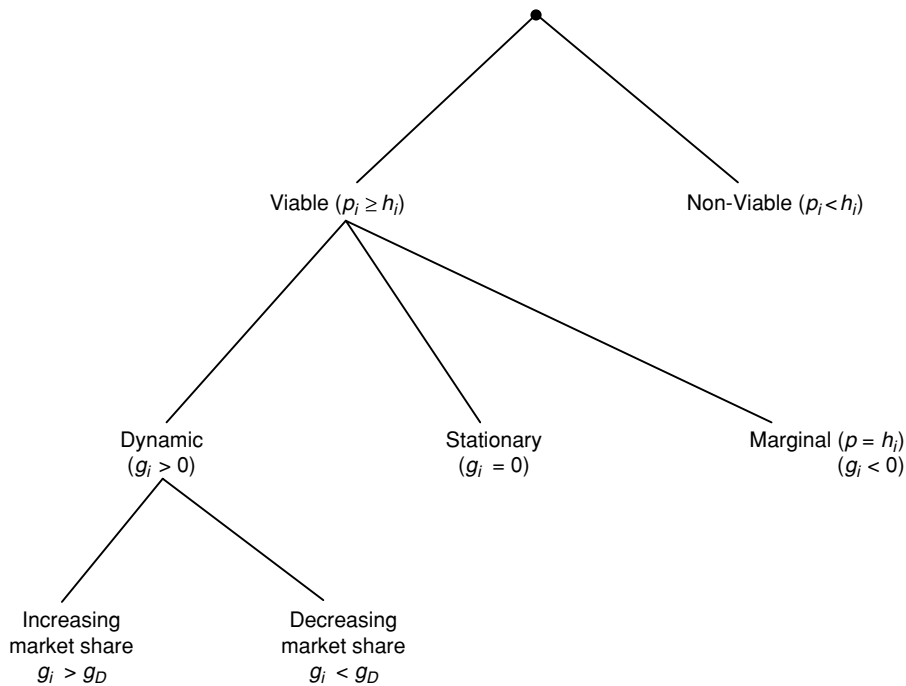


Figure 1.

forward cannot further reduce their prices without incurring losses. The appropriate treatment of marginal firms is not immediately apparent, since as they decline the ensuing excess capacity and associated capital charges will raise unit costs, contrary to our initial hypothesis of constant costs throughout the population. To circumvent this complication, we simplify matters and assume that no firms exist at the static margin defined by $p_i = h_i$.

At an even finer level we can partition the dynamic group of firms according to whether they are increasing their market share ($g_i > g_M$) or not. Clearly, there are interesting boundary cases as well, when, $g_i = g_M$, for example, and we make use of this below. Over its notional lifetime, we may expect any one firm to make a transition between the various states, while at any time the industry consists of firms operating in all the possible states shown in Figure 1.

We explore now the properties of the market selection process implied by these assumptions. First, we establish the pattern of normal prices and thus the pattern of profitability across the industry. Next, the pattern of profitability is related to the growth rates of the competing firms. Finally, because differences in growth rates translate into changes in relative market importance we can establish how the relative importance of the different firms and thus the measure of average performance evolve over time. The point to remember is that the patterns of prices, growth rates and market shares are always changing; in this evolving industry, the market order is always in flux and must remain so as long as firms operating different routines coexist.

Normal prices and growth

We begin by equating (1) and (2), to establish the normal price for a dynamic firm as follows

$$p_i = \frac{1}{\mu + \delta} \{g_M + \phi_i + \delta \bar{p} + \mu h_i\}$$

The average market price \bar{p} is the weighted average of the average prices set by the dynamic firms, $\bar{p}_d = \sum d_i p_i$, and those set by the firms that are stationary, $\bar{p}_s = \sum s_j p_j$. Here d_i is the market share of a firm within the dynamic group and s_j is the corresponding share of a firm within the stationary group. If e is defined as the fraction of total output produced by the stationary firms then, \bar{p} , \bar{p}_d and \bar{p}_s are related by

$$\bar{p} = (1 - e)\bar{p}_d + e\bar{p}_s$$

For any stationary firm, j , its capacity growth rate g_j is by definition zero, so that the investment rule no longer influences its price setting decision, and hence from (2) it must set a price equal to

$$p_j = \frac{g_M}{\delta} + \bar{p} \tag{3}$$

Hence each stationary firms sets the same price, irrespective of its unit costs, and, $p_j = \bar{p}_s$. Correspondingly by summing (1) over the group of dynamic firms we have

$$\bar{p}_d = \frac{g_d}{\mu} + \frac{\bar{\phi}_d}{\mu} + \bar{h}_d \quad (4)$$

Where, $g_d = \sum d_i g_i$, $\bar{h}_d = \sum d_i h_i$ and $\bar{\phi}_d = \sum d_i \phi_i$, are the population averages of the growth rate of unit costs and of the propensity to invest in the dynamic group.

Combining together the average prices for the dynamic and stationary firms yields the value

$$\bar{p} = g_M \left\{ \frac{\delta + e\mu}{(1-e)\delta\mu} \right\} + \frac{\bar{\phi}_d}{\mu} + \bar{h}_d \quad (5)$$

This average price across the whole population increases with the growth rate of the market, it is independent of the unit costs of the stationary firms, and it increases with the share of the output of the stationary firms in total output.

Finally, the difference in the average prices set by the dynamic and the stationary firms is given by

$$\bar{p}_s - \bar{p}_d = \frac{g_M}{\delta(1-e)} = \frac{g_d}{\delta} > 0 \quad (6)$$

since $g_M = (1-e)g_d + eg_s$, and g_s is by construction equal to zero. Notice how the relation between the different average prices depends on the composition of the population in terms of the partitioning between stationary and dynamic firms.

It follows that the normal price for a dynamic firm can be expressed as

$$p_i = \frac{g_d}{\mu} + \frac{\delta}{\mu(\mu + \delta)} \bar{\phi}_d + \frac{1}{\mu + \delta} \phi_i + \frac{\delta}{\mu + \delta} \bar{h}_d + \frac{\mu}{\mu + \delta} h_i \quad (7)$$

The prices defined by (3) and (7) ensure not only that each dynamic and stationary firm is co-ordinated with its own market but that the aggregate growth rate of capacity is also co-ordinated with the overall market growth rate, g_M . These normal prices are set by firms (and not by the market) taking account of the prices set by rivals. They are prices that reflect a degree of market power, they are prices set in excess of marginal costs, and they are not prices associated with perfect competition. Notice that the prices set by dynamic firms depend not only on the selective characteristics of each firm, ϕ_i and h_i , but also on the corresponding population averages of these characteristics. Consequently, it will be helpful to represent the set of normal prices for the dynamic firms by expressing them as deviations from their population average, thus

$$p_i - \bar{p}_d = \frac{1}{\mu + \delta} [\phi_i - \bar{\phi}_d] + \frac{\mu}{\mu + \delta} [h_i - \bar{h}_d] \quad (8)$$

This captures the essential point that the firm which sets a price higher than average, is likely to have singly or in combination either unit costs that are higher than average or a minimum profitability requirement that is higher than average. By virtue of the assumption of constant returns to scale, this distribution of prices is independent of the market growth rate. Notice also how the characteristics of the market environment, δ , μ , translate the deviations in firm characteristics, ϕ_i , h_i , into deviations in normal prices around the respective population means. Clearly, if $\delta = \infty$, then $p_i = \bar{p}_d = p$, for all the firms. By contrast, when $\delta = 0$, every firm grows at the common, market rate g_M and its normal price follows from (1). If $\mu = \infty$, then $p_i = h_i$. If $\mu = 0$, however, normal prices cannot be defined, growth cannot be financed and the competitive process breaks down.

Growth rates and market share dynamics

From the normal prices we can immediately derive the normal growth rates that keep capacity growth in line with demand growth for each dynamic firm. These are given by the distance from population mean formula

$$g_i = \frac{g_M}{1 - e} + \frac{\mu\delta}{\mu + \delta} [\bar{h}_d - h_i] + \frac{\delta}{\mu + \delta} [\bar{\phi}_d - \phi_i] \quad (9)$$

so that deviations around the average growth rate of the dynamic group are combinations of the corresponding deviations of ϕ_i and h_i around their population average values. Now the wider significance of (9), is that it defines a replicator dynamic process. The difference, $g_i - g_d$, measures the proportionate rates of change of the market share of firm i within the dynamic group, hence,

$$\frac{d(d_i)}{dt} = d_i(g_i - g_d) \quad (9')$$

The pattern of market shares at each instant in time captures the order in the market, the order that arises from the normal process of co-ordination, and this order changes in a very precise way. Within the population of dynamic firms, it changes according to the replicator relation (9'). However, the changing market order is not only with respect to the dynamic firms. Since the stationary firms are exactly that, it follows that their share of the market declines towards zero as the growth rate of the dynamic firms converges on the market growth rate. Indeed the rate of change of e , is given by

$$\frac{de}{dt} = -eg_M \quad (9'')$$

as the reader can readily check.

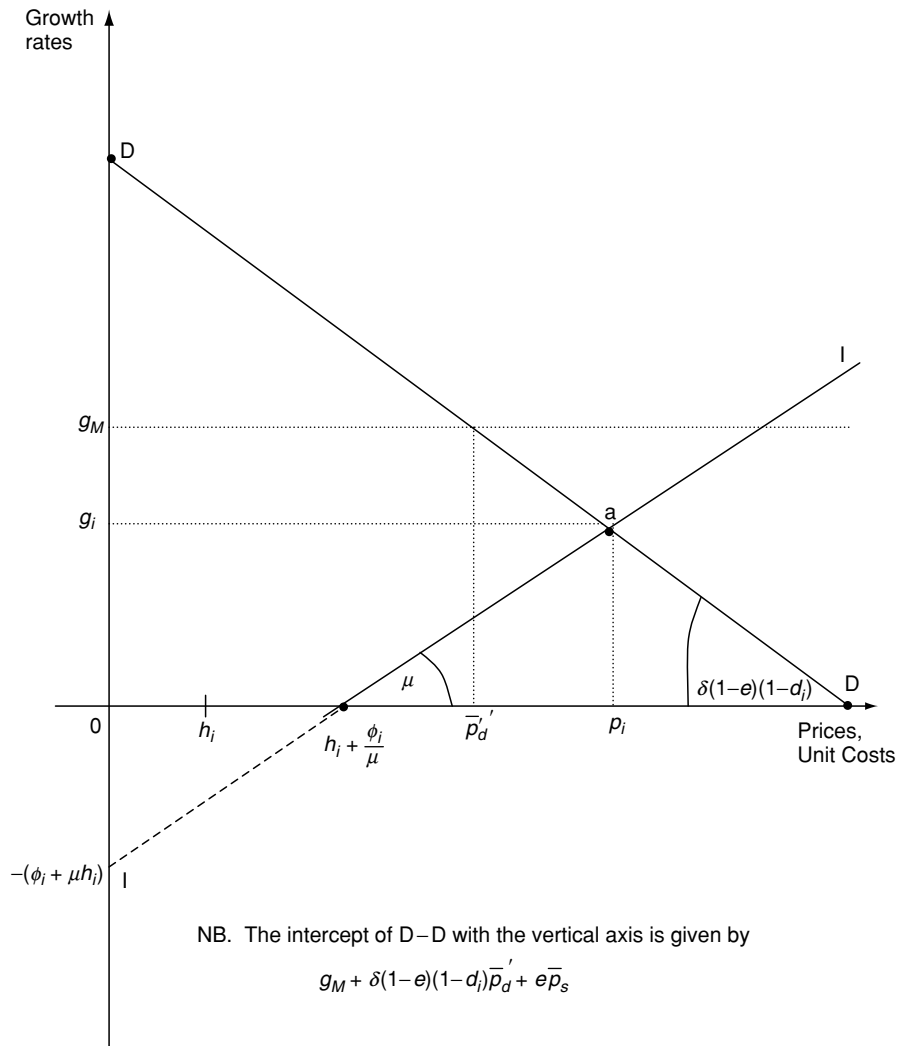


Figure 2.

A diagrammatic treatment

Figure 2 provides a concise way of summarizing all of the relations implicit in the pricing and investment behavior of a particular dynamic firm³ and helps identify the various states identified in figure 1. The schedule $D - D$ is the particular dynamic demand relation for the firm (2) and $I - I$ is its investment routine (1). The level of unit cost is h_i . As drawn, this firm is dynamically co-ordinated at point 'a' and it sets normal price p_i and grows at rate g_i . This growth rate is less than the aggregate growth

rate g_d and the price is greater than the average price \bar{p}'_d . Clearly, this firm is dynamic, it is profitable but it is losing market share to some of its dynamic rivals.

How would a stationary firm represented in figure 2? Its demand schedule would intersect the horizontal axis to the left of the point where its particular investment relation cuts that axis but to the right of its particular unit cost, h_i . The values of h_i and ϕ_i that define the margin separating growth and stationarity are given by setting (9) equal to zero, thus

$$h_i = \frac{g_M}{\Delta_1(1-e)} + \bar{h}_d + \frac{\Delta_2}{\Delta_1}(\bar{\phi}_d - \phi_i) \quad (10)$$

where $\Delta_1 = \frac{\delta\mu}{\delta+\mu}$ and $\Delta_2 = \frac{\delta}{\delta+\mu}$ are the market selection coefficients defined by the properties of the product market and the capital markets.

The margin separating stationarity from viability is obviously reached when the particular demand relation cuts the horizontal axis at the associated value, h_i , of that firm's unit costs. The point of co-ordination in Figure 2 is of course transient, the particular demand schedule $D - D$ is always changing as market shares evolve. The pattern of order there shown is restless; we associate evolution with order not with equilibrium.

The selection set

To conclude this section, Figure 3 provides a useful way to illustrate the forces of variation and selection in this population. Two ideas are combined together in this diagram. The first is that of the selection set. This depicts the given population of all firms that operate or have operated in the past, each firm characterized by the parameters, ϕ_i , h_i . This set is represented by the convex hull, labeled $n - n$ in the diagram. Nothing in the evolutionary argument hinges on this space being filled in a uniform fashion by the firms. It may contain 'holes' and, indeed, it may be empty in the interior, the only firms being those identified with the label n . How this set has been constructed reflects contingent historical circumstances of innovation and the entry of firms, precisely those development issues that we have suppressed.

The second idea is that of an economic partitioning of this set along the lines of figure 1 and the identification of the economic attributes of the corresponding regions of the selection set.

Consider the line $A - A$. This is derived from (9). It shows all the combinations of ϕ_i and h_i at which a dynamic firm will grow at the group average rate, $g_d = \frac{g_M}{1-e}$. It passes through the point defined by the population means and its slope is equal to $\Delta_2/\Delta_1 = 1/\mu$, which is independent of δ . A firm located on this line has a neutral competitive advantage among the dynamic firms, it is statistically representative of the dynamic group as a whole and its market share within this group will be constant. Consequently, any firm, such as β , located in the set below $A - A$ will have a competitive advantage and will grow at a rate greater than the dynamic group as a whole. Since all the firms in this shaded region below $A - A$ have growth rates that exceed the growth

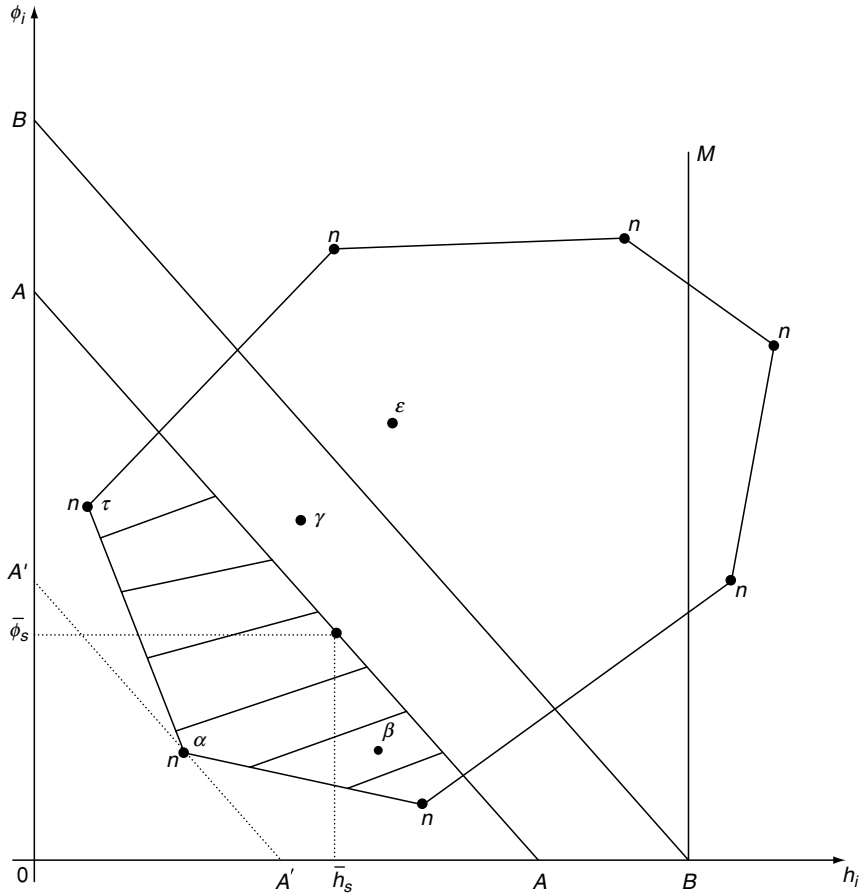


Figure 3.

rates of firms that lie above $A - A$, it is obvious that the selection dynamic must be drawing this locus towards the origin. Since all the firms that are located on $A - A$ have the same growth rate, we call it an ‘iso-growth’ line. Its significant attribute is that the slope indicates the willingness of the capital market to fund the dynamic firms.

Consider next the iso-growth line labeled $B - B$. This is derived by setting $g_i = 0$ in (9) to show all those combinations of ϕ_i and h_i at which a firm has reached the margin of stationarity (10). All firms on $B - B$ have ceased to invest but only just. Consequently, the dynamic group of firms is contained in the portion of the selection set below $B - B$. This leaves two spaces to consider.⁴ To identify these we first establish the viability margin, this is given by the vertical locus labeled $M - B$. This locus is defined by the condition $p_i = h_i$ for an already stationary firm and at this margin

$$h_i = \frac{g_M}{\delta} + \bar{p}$$

Making use of the definition of \bar{p} in (5), this becomes

$$h_i = \frac{g_M}{\Delta_1(1-e)} + \bar{h}_d + \frac{\Delta_2}{\Delta_1}(\bar{\phi}_d) \quad (11)$$

Consequently, the unit cost value that defines the viability margin also coincides with the unit cost value of a particular stationary firm, that firm with $\phi_i = 0$ in (10). Any firm to the right of $M - B$ is not viable and we have assumed it has ceased operation. This leaves the segment of the selection set bounded to the right by $M - B$ and below by $B - B$, which defines the region of stationarity. Thus firm ε is profitable but stationary, while firm ν is dynamic but losing market share in the dynamic group.

Just as the selective characteristics of the firms differ so do their economic characteristics and, as always, an evolutionary process depends on variety, in this case, on the co-existence of firms of different economic kinds.⁵

Evaluating the optimality of the competitive process

We have obtained a complete description of the competitive process from the discussion in section 2. This provides the means to evaluate the three propositions set out at the beginning of this paper, and we begin with the most straightforward case, namely, proposition A.

Proposition A

To evaluate proposition A, it is sufficient to enquire what will happen if this competitive process is allowed notionally to run its course with an invariant selection environment and without any changes in respect of the characteristics of the firms in the selection set? In these implausible conditions, the answer is clear and is captured in Figure 4. Selection seeks out the most dynamic of the firms, the one with the highest growth rate. The market share of this firm tends to unity, while the values of $\bar{\phi}_d$ and \bar{h}_d converge on the values of ϕ_i and h_i for this optimum firm. Is this 'optimum' firm the firm with the lowest unit costs as Proposition A requires? From Figure 4 it is obvious that this is not necessarily so, the outcome depends on the contingent shape of the selection set and the properties of the evaluating environment as captured by the capital market parameter μ . The firm that is 'finally' selected in Figure 4 is firm α , which is identified by the point on the boundary of the selection set supported by the locus $A' - A'$. Firm α , unlike τ , is not the least cost firm.

Thus, in general, Proposition A fails. Competition does not in general, and in the long run, concentrate the resources deployed in the industry on the most efficient firm. All we can establish is that the competitive process drives the industry to the boundary of its selection set, this is the proper sense in which the limit of the competitive process is optimal. Of course, if all firms had the same propensity to invest, Proposition A would

has also dropped to zero with time. Obviously, all firms in the region labeled N are out of business. As with any evolutionary argument, survival must be distinguished from market significance.

How does the capital market influence this outcome? If the supply of finance places no constraint on the rate of growth of firms, $\mu = \infty$, then $A' - A'$ becomes the vertical line $A'' - A''$. The optimal firm is now firm τ , that firm with the lowest unit costs in the selection set. Differences in ϕ_i across the firms become irrelevant to the selection process so τ dominates and, indeed, it is the only long-run survivor in the selection set, all other firms are eventually eliminated.⁷ There is another aspect of this case worth noting. If δ is also ∞ , it follows that the market would select firm τ instantly, whatever the distribution of firms in the selection set. Thus, the validity of proposition A depends on there being no limitations on the supply of finance. More generally, the less readily is finance available the less likely is it that the least cost firm will be selected in the long run.⁸

In providing a precise evaluation of proposition A, we have necessarily made the argument artificial. There is no reason to expect that a selection set for any industry will remain constant over time or that it will be evaluated by a constant environment. Firm α may be selected in one set of circumstances but not another. There is no sense in which it is the optimum firm other than contingently. Even within these limitations however proposition A is not valid in general, and it is the tenuous nature of the long-run argument that makes proposition B interesting.

Proposition B

Unlike proposition A, the second proposition concerns the dynamics of the competitive process. Will competition always increase the average efficiency with which resources are utilised? In our framework, this is equivalent to the question, 'Do more efficient firms always grow more rapidly than their less efficient rivals and account for a greater proportion of the resources utilised in the industry?'. To answer this question, it is sufficient to focus only on the dynamic firms.⁹

Equation (9) provides us with ' n ' equations for the rates of change of market shares among the dynamic group and, because market shares are changing, it follows that the average population characteristics $\bar{\phi}_d$ and \bar{h}_d must also be changing, and with them the distribution of prices and growth rates. This population of firms is always co-ordinated but is never in equilibrium, it is always restless.

It follows directly from (9) that

$$\frac{d\bar{h}_d}{dt} = \frac{d(d_i)}{dt} h_i = \sum d_i (g_i - g_d) h_i = -[\Delta_1 V_d(h) + \Delta_2 C_d(\phi, h)] \quad (12)$$

Where $V_d(h) = \sum d_i (h_i - \bar{h}_d)^2$ and, $C_d(\phi, h) = \sum d_i (\phi_i - \phi_s) (h_i - \bar{h}_d)$.

This is the result that I have elsewhere called Fisher's Principle (Metcalf 1998, Frank 1998). It relates the rate of change in the population mean of a given

selection characteristic to the population variance of that characteristic and the population covariance between that characteristic and other selection characteristics.¹⁰ It captures the essential ‘distance from mean dynamic’ (9) that defines a replicator process. As in all variation, selection processes, how average behavior changes depends on the joint distribution of all the selection characteristics in a population *and* it depends also on the nature of the selection environment as reflected in the selection coefficients.

We can see immediately from (12), that proposition B is in not universally true. If the covariance between unit cost and propensity to invest is negative and sufficiently large then average unit cost will increase and the average efficiency of resource utilization will decline. Competition does not increase efficiency automatically the outcome is contingent. Conversely, if lower cost firms also have a greater propensity to grow then Proposition B will certainly be valid. In two further special cases, the proposition is valid. The first, trivial, case is again if all the firms have the same propensity to invest for then the covariance is automatically zero. This is equivalent to making unit cost the only criterion for selection; there are no interfering effects from other characteristics of the firms. The second case is when μ is infinite and the supply of finance is not a constraint on the growth of firms. Then (12) reduces to

$$\frac{d\bar{h}_d}{dt} = -\delta V_d(h)$$

The effects of different propensities to accumulate are negated, and the rate of selection depends only on the efficiency of the product market and the variance in unit costs. By a similar argument, we deduce that the average propensity to invest evolves as

$$\frac{d\bar{\phi}_d}{dt} = -[\Delta_2 V_d(\phi) + \Delta_1 C_d(\phi, h)] \quad (13)$$

If we now combine (12) and (13), we recover a variant of proposition B that is generally valid. Namely, that the weighted sum of the rates of change in the two population means satisfies the following conservation principle.¹¹

$$\left\{ \Delta_1 \frac{d\bar{h}_d}{dt} + \Delta_2 \frac{d\bar{\phi}_d}{dt} \right\} = -V_d(g) < 0 \quad (14)$$

The weighted sum of the change in population means is always negative; at least one of the population means must be declining but not necessarily both of them. As we have already established, selection may increase or decrease \bar{h}_d . Indeed competition seems to imply this conservation condition across a wide variety of evolutionary models. On average, selection always works to enhance the potential for growth in the population as a whole, that is to say, it always increases average economic fitness. This brings the argument conveniently to proposition C and Kimura’s theorem.

Proposition C

This theorem is more subtle than its companions, A and B. It asks questions of the exact form of the competitive process in our industry, for there are many alternative ways in which competition can reallocate resources than the way embodied by the replicator dynamic, (9). Each alternative will be associated with a specific pattern of changes in market shares of the firms and thus a particular evolution of the population averages of the selective characteristics in relation to unit costs and propensities to accumulate. The force of Kimura's theorem is that all competitive processes that do not support a particular replicator process are inferior, in the sense that they do not maximize the rate of change of these population averages.

Kimura's original statement of his theorem was applied to fitness values, defined as the growth rates of entities in a population. In the context of the link between efficiency and competition, it is more instructive to apply the theorem to one of the determinants of fitness, namely unit costs. The issue to be settled is whether the competitive process described above produces the maximal rate of change in the average efficiency with which resources are utilized in the industry/market. That is equivalent to producing the maximum rate of change in average unit cost as defined above. To keep the argument as clear as possible we shall assume that the relevant industry consists only of dynamic firms, those firms that are sufficiently profitable to invest in capacity expansion.

The Kimura theorem

Consider a population of 'n' firms each one growing in scale at a given compound rate, g_i . The aggregate growth rate of the population is $g_d = \sum d_i g_i$, where d_i is the relative importance (market share) of a firm in the population. If $x_i(t)$ is the output of that firm, then $d_i(t) = x_i(t) / \sum x_i(t)$, the sum being taken over all the dynamic firms in the population.

Consider small intervals of time of length θt and index each time interval by its end date. Between two intervals, we imagine that each market share changes by the (different) increments θd_i . Define growth rates, one for each entity, $g_i(t + \theta t)$, such that

$$g_i(t + \theta t) \cdot \theta t = \frac{x_i(t + \theta t)}{x_i(t)} - 1$$

Then it follows that

$$\theta d_i = d_i(t) \left\{ \frac{g_i(t + \theta t) - g_d(t + \theta t)}{1 + g_d(t + \theta t)\theta t} \right\} \theta t \quad (15)$$

Where, $g_d(t + \theta t) = \sum d_i(t) g_i(t + \theta t)$ is the aggregate rate of growth for the entities in the population between the two intervals $t + \theta t$ and t . Notice that the growth rates are weighted by the shares in the previous interval.

Clearly, it follows that

$$\sum \theta d_i = 0 \quad (16)$$

Less obviously, upon squaring both sides of (15) and summing across the population, we find that

$$\sum \frac{1}{d_i(t)} (\theta d_i)^2 = \frac{V_d(g)(\theta t)^2}{(1 + g_d(t + \theta t)\theta t)^2} \quad (17)$$

Where, $V_d(g) = \sum d_i(t)[g_i(t + \theta t) - g_d(t + \theta t)]^2$ is the weighted variance of the growth rates of the firms.

Expressions (16) and (17) define two independent aggregation constraints on the permissible variations in the values θd_i one for each firm.¹² They apply whatever the process that generates the evolution of the market shares. Indeed, of the possible competitive processes, that embodied in (9) is only one of many candidates and this observation leads directly to the Kimura theorem. The Kimura theorem asks the question ‘Which pattern of changes in the ‘n’ market shares will maximize the rate of change of the population average, \bar{h}_d , subject to the constraints (16) and (17)?’. This leads to a second question, ‘Does the particular competitive process embodied in (9) above follow such an optimal rule?’. The answer to this question is to the affirmative but to establish this claim requires a little thought. Rather than work directly with the original values for each firm’s unit cost, it is more convenient to proceed in an elliptical fashion that greatly simplifies the proof we seek.

We have established that if we take two firms with identical unit costs but different propensities to accumulate, the firm with the larger absolute value of ϕ_i will have the lower rate of growth. We can interpret its inferior propensity to accumulate as equivalent to a unit cost penalty; and we measure this penalty by asking how much smaller its unit costs must be to offset the consequences for its growth rate of the inferior propensity to accumulate. This suggests a general method of analysis, namely, to represent each firm in terms of a hypothetical unit cost level, h_i^* , that is greater or less than the true cost level h_i as ϕ_i is greater or less than $\bar{\phi}_d$. We will call these hypothetical unit costs the dynamic cost equivalents and their value for any firm is given by

$$h_i^* = h_i + \frac{1}{\mu} (\phi_i - \bar{\phi}_d) \quad (18)$$

In effect, we have created a dynamically equivalent ‘win’ for each firm, the two always growing at the same rate but with the twin, virtual equivalent having unit costs equal to h_i^* and a value of $\phi_i = \bar{\phi}_d$. Since they are dynamically equivalent, the ‘twin’ has exactly the same impact on the competitive process and the evolution of market shares as the original firm. In this way, we can reduce the two dimensional distribution of characteristics to a one-dimensional distribution of the virtual cost values h_i^* .¹³ Using

the dynamic cost equivalent representation of each firm, we can greatly simplify the derivation of Kimura's theorem and so evaluate the validity of Proposition C for our particular competitive process. This follows because on summing the values of the dynamic cost equivalents for the 'twins' we find that $\bar{h}_d^* = \bar{h}_d$, the average unit cost in the true population.

It follows that

$$\frac{d\bar{h}_d}{dt} = \frac{d\bar{h}_d^*}{dt} \quad (19)$$

and we can legitimately evaluate proposition C by applying the Kimura method to the evolution of the twin virtual population average \bar{h}_d^* . The procedure is as follows. We are to maximize $-\theta\bar{h}_d^*$ subject to the constraints on the θd_i values provided by (16) and (17). Thus the function to be maximized is

$$L = - \sum \theta d_i h_i^* - \sum d_i \theta h_i^* - \sum \theta d_i \theta h_i^* \\ + \lambda_1 \left[\sum \frac{\theta d_i^2}{d_i} - \frac{V_d(g)\theta t}{1 + g_d \theta t} \right] + \lambda_2 \sum \theta d_i$$

The first and second order conditions are

$$\frac{\partial L}{\partial(\theta d_i)} = -h_i^* - \theta h_i^* + \frac{2\lambda_1 \theta d_i}{d_i} + \lambda_2 = 0 \quad i = 1 \dots n$$

and

$$\frac{\partial^2 L}{\partial(\theta d_i)^2} = \frac{2\lambda_1}{d_i} < 0$$

for a maximum.

Weighting each first order condition by that firm's market share and summing gives $\lambda_2 = \bar{h}_d^* + \sum d_i \theta h_i^*$ and so at the optimum

$$\frac{2\lambda_1 \theta d_i}{d_i} = (h_i^* - \bar{h}_d^*) + (\theta h_i^* - \sum d_i \theta h_i^*)$$

Noting that $\theta h_i^* = (1/\mu)\theta\bar{\phi}_d$ is the same for all firms, from (18), this reduces to the conditions

$$\frac{2\lambda_1 \theta d_i}{d_i} = (h_i^* - \bar{h}_d^*), \quad i = 1 \dots n$$

The procedure for eliminating λ_1 is to square both sides of the first order condition and sum across the population, while taking account of (17), to give

$$\frac{1}{2\lambda_1} = \pm \sqrt{\frac{V_d(g)}{V_d(h^*)}} \frac{\theta t}{1 + g_d \theta t}$$

Thus since $\lambda_1 < 0$ at a maximum, the optimal rule for the evolution of the market shares is,

$$\frac{\theta d_i}{\theta t} = \left(\frac{-\Delta_1}{1 + g_d \theta t} \right) d_i (h_i^* - \bar{h}_d^*)$$

and, as we let θt tend to zero, this becomes

$$\frac{d(d_i)}{dt} = \Delta_1 d_i (\bar{h}_d^* - h_i^*) \quad (20)$$

This is the rule for the evolution of the ‘n’ market shares that maximizes the rate of change of the average unit cost level.

However, from the definition (18) of the unit costs of the ‘twin’ firms we find that

$$\Delta_1 (h_i^* - \bar{h}_d^*) = \Delta_1 (h_i - \bar{h}_d) + \Delta_2 (\phi_i - \phi_d),$$

This means that (20) is identical to (9), and we can conclude that the competitive process embodied in (9) has the effect of redistributing market shares over time to maximize the rate of reduction of unit average costs. Along this ‘Kimura path’, we find that

$$\sum \frac{d(d_i)}{dt} h_i^* = -\sqrt{V_d(g)V_d(h^*)} < 0$$

and consequently that

$$\frac{d\bar{h}_d^*}{dt} = -\sqrt{V_d(g)V_d(h^*)} + \frac{1}{\mu} \frac{d\bar{\phi}_d}{dt} = \frac{d\bar{h}_d}{dt}$$

It follows from this that Proposition C is valid, since the competitive process maximizes the rate of change of the virtual population average, \bar{h}_d^* , it necessarily maximizes the rate of change of the true population average. Notice carefully that this means that optimization in the sense of Proposition C is a system attribute not a characteristic of the individual firms. Whether or not the individual firms act according to maximizing rules, it is the market process, which is the maximizing agency in validating Proposition C, not the individual firms.

By applying exactly the same method to the average propensity to accumulate, we can show that (9) also maximizes the rate of change of $\bar{\phi}_d$. Consequently, since the selection process maximizes the rate of change of the left hand side of (14) it also maximizes the variance of the growth rates. Given the distribution of the selective characteristics of the population of firms, the competitive rules embodied in (1) and (2) generate paths for the market shares that keep the variance in growth rates as large as is

possible. In reaching this conclusion, we make precise a cardinal rule of evolution; that its rate depends upon the variation of fitness in the population.¹⁴

This application of Kimura's method has uncovered an optimizing principle that is apparently unknown in the competition literature. If the rules governing the competitive dynamic are not based on the Fisher Principle, replicator rules of the kind embodied in (20), then the rate of change of average efficiency will be lower than it could otherwise be. Market institutions that support a replicator dynamic of this precise kind are in this sense optimal.¹⁵

However, given the particular replicator dynamic (9), it also follows that the rate at which unit costs change on the optimal path depends upon the nature of the selection environment. A belief that competition always and everywhere supports all three of our propositions now falls into place. A perfect product market ($\delta = \infty$) and an infinitely elastic supply of finance ($\mu = \infty$), taken together, are necessary and sufficient for the competitive process to concentrate all the resources of the industry on the most efficient firm immediately; in this case the replicator process operates instantaneously. Indeed, where these two assumptions hold the model industry would always and without exception be at its best possible long run position. In short, all superior business rules are adopted in full, instantaneously. The case for the evolutionary argument is thus the case that the world is not like this, even approximately, and that it takes time for competition to work itself through the prevailing population.

Of course, we have not addressed the question of the existence of alternative competitive processes, with rules that could be dynamically superior to those embodied in (9). Rather we have shown that, conditional on the distribution of selective characteristics, the competitive process that leads to (9) will have the effect of maximizing the rate of change of the population averages for those selective characteristics. Moreover as we notionally vary the properties of the selection environment, so we vary the characteristics of the optimal market share paths.

Conclusion

The possible optimality properties of market processes are not a trivial matter since the normative appraisal of capitalism is frequently based on the claim that 'markets maximize'. We have evaluated this claim in terms of three independent propositions and found that two of them are not in general valid. Competition does not, in general and in the long run, select the most efficient firm or eliminate all the inefficient firms in an industry. Nor does a process of competitive selection ensure that the fastest growing firms are always the most efficient firms, so that competition does not always increase the average efficiency of the utilization of resources. We have established, however, that competitive selection maximizes the rate at which the average efficiency in an industry changes over time.

In the light of this, is there any warrant for the general view that competition optimises the allocation of resources? A more nuanced appraisal of the institutions of the competitive process is surely necessary. From an evolutionary standpoint, the

outcomes of competition are always contingent on the nature of the selection environment and the characteristics of the whole population of firms that are being selected. All that we have needed to establish these conclusions is the recognition that the competitive performance of firms is multi-dimensional, and that a theory of competition requires a treatment of firm expansion as well as firm efficiency in the traditional sense (Winter 1963, p. 244).

As pointed out previously this remains only half an argument. The case for market capitalism rests more fundamentally on its properties in relation to the innovation process and the pursuit of imaginative business conjectures. This ‘second half of the argument’ we have not addressed, for it requires a fully articulated theory of innovation. Although, perhaps it is as well to get the first half of the argument straight before attempting this far greater task.

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Notes

1. To avoid misunderstanding, this is not the same work as that for which Kimura is more widely known, namely his neutral evolution hypothesis.
2. Since these normal prices have the property that each firm whether growing or stationary maintains full capacity utilisation, these prices could be interpreted as long-period normal prices in the Marshallian sense.
3. In drawing this diagram we have made use of the fact that the average price in the market includes the price of this particular firm, and we have separated this out by defining \bar{p}'_d as the average price of all the firms excluding the one that is in focus. Thus we have used the relations $\bar{p} = (1 - e)\bar{p}_d + e\bar{p}_i$ and $\bar{p}_d = d_i\bar{p}_i + (1 - d_i)\bar{p}'_d$ in drawing figure 2. Figure 2 is a useful device to change parameter values and investigate the consequences but we leave this to the reader.
4. We may note that between $A - A$ and $B - B$ there is a third iso-growth line derived by setting the growth rate of the firm equal to g_M . We should note that a firm that is losing market share in the dynamic group could still be increasing its share of the overall market if

$$g_d > g_i > g_M \quad \text{that is if} \quad g_d > g_i > g_d(1 - e)$$

5. We leave it to the interested reader to explore how the partitioning of the selection set is sensitive to the rate of growth of the market. If the growth rate is zero for example, then the loci $A - A$ and $B - B$ coincide. In

general, the faster the rate of growth of the market, the less is the degree of competitive pressure on the margins of any one firm.

6. This we can see in figure 4 by noting that the share of the stationary firms has been driven to zero and has redefined the lines $B' - B'$ and $M' - B'$.
7. The lines $B' - B'$ and $M' - B'$ now coincide with $A'' - A''$.
8. Notice too that this conclusion does not depend upon capital market imperfections for the capital market treats all the firms equally.
9. Note that average unit cost and thus efficiency in the stationary firms is constant. If \bar{h} is defined as average unit cost across all the operating firms it follows that

$$\frac{d\bar{h}}{dt} = (1 - e) \frac{d\bar{h}_d}{dt} - e g_M (\bar{h}_s - \bar{h}_d) < 0$$

defines the general condition for \bar{h} to decline over time.

10. Frank (1998) contains an excellent account of Fisher's theory of selection.
11. Notice carefully that in deriving (11), (12) and (13) all of the statistical moments are 'weighted moments', they are constructed using the market share weights d_i . Consequently, these higher moments evolve too (Metcalf, 1998).
12. In what follows it will be convenient to drop the time subscripts, the correct phasing can always be recovered from (15).
13. Of course, the twin unit costs are not constants, they vary with the variation in the average propensity to accumulate.
14. The reader can check this by maximising the sum, $-(\Delta_1 \theta \bar{h}_d + \Delta_2 \theta \bar{\phi}_d)$ subject to (16) and (17) and finding that the optimal rule is given again by (9).
15. Consider an alternative 'selection rule', for example, that given by $d(d_i)/dt = -[\Delta_1 u_i + \Delta_2 w_i]$ where u_i and w_i are random numbers with zero means that are subject only to the constraint that $-\sum[\Delta_1 u_i + \Delta_2 w_i] = 0$. It is obvious that this rule does not give the maximum rate of change of average unit costs. Indeed, if the values of u_i and w_i are statistically independent of the values of h_i

$$\frac{d\bar{h}_d}{dt} = -[\Delta_1 C_d(h, u) + \Delta_2 C_d(h, w)] = 0$$

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